Automatic Regulation Time Series for Sampled-data Feedback Control Systems

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Abstract

In this literature, a new nonlinear digital controller is proposed for analyses and designs of sampled-data feedback control systems. The controller is derived from the converging characteristic of a specified numerical series. The ratios of neighborhoods of the series are formulated as function of the output of the plant and the reference input command, and will be converged to be unities after the output has tracked the reference input command. Two kind of servo system examples are used to illustrate effectiveness of the proposed nonlinear digital controller.

Keywords: Time Series, Nonlinear Control, feedback control system.

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I. Introduction

For unit feedback discrete-time control systems, the control sequences are usually functions of the difference between the sampled reference input and output of the plant [1-4]. The discrete-time control sequence can be generated by Finite Impulse Response (FIR) filter or Infinite Impulse Response (IIR) filter. The input of FIR or IIR filter is the difference between the sampled reference input and output of the plant. The output of FIR or IIR will be the input of the plant. In general, they are linear controllers. In this literature, a nonlinear discrete-time control sequence described by periodic numerical series $G(jT_S)$ with ratios of the reference input and plant output is first proposed for analyses and designs of sampled-data feedback control systems. T_S represents the sampling interval. The ratios of $G((k+1)T_S)$ to $G(kT_S)$ of the series are formulated as a function of the reference input command and the output of the plant. The value of $G(kT_S)$ is the control input of the plant at time interval between $(k-1)T_S$ and kT_S. Thus, the considered system is closed with $G(jT_S)$. It will be seen that the output of the plant tracks the reference input command after ratios $G((k+1)T_S)$ / $G(kT_S)$ of the series being converged to unities. It implies that $G(kT_S)$ will be converged to a steady-state value for a constant reference input applied. The stability of the closed-loop system is guaranteed by selecting the proper function of ratios $G((k+1)T_S)/G(kT_S)$. It will be proven that the considered system with $G(kT_S)$ becomes a negative feedback control system.

In following sections, basic concepts of the proposed nonlinear discrete-time control sequence are discussed first, and then two servo system examples are used to illustrate their tracking behaviors. Simulating results will show that the proposed nonlinear digital controller gives another effective way for analyses and designs of sampled-data feedback control systems.

II. The Automatic Regulation Time Series

A numerical series with time interval T_S [1-4] can be written as in the form of

$$G(jT_S),\ j = 1, 2, 3, \ldots, n, n+1, \ldots\ldots\ldots \quad (1)$$

where $G(jT_S)$ represents a constant value between time interval between $(j-1)T_S$ and $j\,T_S$. For simplicity, the representation of $G(jT_S)$ will be replaced by $G(j)$ in following evaluations. The ratios $G(j+1)/G(j)$ of the series are defined as in the form of

$$F(j) = G(j+1)/G(j),\ j= 1,2,3,., n, n+1,.. \quad (2)$$

Equation (2) shows the value of $G(n+1)$ approaches to be a constant value when the value of $F(n)$ approaches to be unity. Now, the problem for closing the considered system is to find the formula of $F(j)$ which is the function of the reference input command R and the output of the plant Y. $G(n+1)$ is used as the input of the considered system. Considering a series given bellows:

$$G(n+1) = \left[\sum_{i=0}^{m} a_i \left(R(n)/Y_S(n)\right)^i\right] G(n) \quad (3)$$

where $R(n)$ represents the reference input command and $Y_S(n)$ represents the non-zero sampled output of the plant at the sampling interval $n\,T_S$. Equation (3) is a possible way to close the considered system as a sampled-data feedback control system. Assume the reference input command has been tracked by applying control effort $G(j)$, Equation (3) becomes

$$G(n+1) = \sum_{i=0}^{m} a_i G(n) \quad (4)$$

For steady-state condition, $G(n+1)$ approaches to be a constant value, it gives

$$\sum_{i=0}^{m} a_i = 1 \quad (5)$$

Rearranging Equation (3) and taking the derivative of it with respect to $Y_S(n)/R(n)$, we have

$$F(n) = \sum_{i=0}^{m} a_i \left(Y_S(n)/R(n)\right)^{-i} \quad (6)$$

And

$$\partial F(n)/\partial(Y_S(n)/R(n)) = -\sum_{i=0}^{m} ia_i\left(Y_S(n)/R(n)\right)^{-i-1} \quad (7)$$

The sufficient but not necessary condition for Equation (7) less than zero is $a_i > 0$ for $Y_S(n)/R(n) \fallingdotseq 1$ and Equation (6) is rewritten as in the form of

$$F(n) = \sum_{i=0}^{m} a_i \left\| Y_S(n)/R(n) \right\|^{-i} \quad (8)$$

$a_i > 0$ will be used in following evaluations. Negative value of Equation (7) represents the closed-loop system with Equation (3) activated as a negative feedback system around the equilibrium condition; i.e., $Y_S(n) = R(n)$. These statements will be illustrated by the first order polynomial described in Equation (3). It is in the form of

$$G(n+1) = \left[(1-\beta)R(n)/Y_S(n) + \beta\right]G(n) \quad (9)$$

where β satisfies constrains stated above and becomes a adjustable parameter. Thus, the ratios $F(n)$ becomes

$$F(n) = (1-\beta)/(Y_S(n)/R(n)) + \beta \quad (10)$$

Taking the derivative of Equation (10) with respect to $Y_S(n) = R(n)$, we have

$$\partial F(n)/\partial(Y_S(n)/R(n)) = -(1-\beta)/(Y_S(n)/R(n))^2 \quad (11)$$

For negative value of Equation (11), the value of β must be less than one. The suitability of the proposed nonlinear adaptive digital controller is based upon this characteristic. Fig.1 shows ratios $F(n)$ versus $Y_S(n) = R(n)$ represented by Equation (8) for β = 0.9, 0.7, 0.5 and 0.3; respectively. Fig. 1 shows that the value of $F(n)$ is less than one for that of $Y_S(n)$ greater than that of $R(n)$, then the value of $G(n+1)$ will be decreased; and the value of $F(n)$ is greater than one for that of $Y_S(n)$ less than that of $R(n)$, the value of $G(n+1)$ will be increased. This implies that the controlled system connected with Equation (8) will be regulated to the equilibrium point ($Y_S(n)/R(n)$ =1) and gives a negative feedback control system for deviation from equilibrium point. From Fig.1, it can be seen that one can adjust β to get desired regulating slope; i.e., regulating characteristic. Certainly, other tracking functions can be formulated and proposed also for the considered system, if its derivative with respect to $Y_S(n)/R(n)$ is negative.

Fig.2 shows the connected system configuration in which Equation (9) and output of the nonlinear controller are modified for negative control swing is generally required. The equation is rewritten as in the form of

$$G(n+1) = [(1-\beta)(R+Y_o)/(Y_S(n)+Y_O)+\beta]G(n) \quad (12)$$

where Y_o represents the negative control swing, $Y_S(n)$ represents the sampled with hold output of the plant at sampling interval nT_S, and U is the sampled with hold output of the controller. The values of $G(n)$ and $F(n)$ will be all positive for the summation of $Y_S(n)$ and Y_o (or R and Y_o) is greater than zero with specified values of Y_o. All positive values will give better continuity and regulating characteristic of the series. The value of β is greater than zero and less than one. Equation (11) implies ratios $G(n+1)/G(n)$ are in the form of

$$F(j) = [(1-\beta)(R+Y_O)/(Y_S(j)+Y_O)+\beta], \quad j = 1,2,3,\cdots,n,n+1,\cdots \quad (13)$$

For illustration purpose, the equivalent block diagram of Fig. 2 is shown in Fig.3, in which small offset values ε given in nonlinearities N_1 and N_2 will be used to prevent numerical singularities of Equations (12) and (13) and null of $G(n+1)$. Note that the zero value of $G(n+1)$ will make values of $G(n+k)$ be equal to zeros for k is greater than one; i.e., the controlled system becomes open. The saturation level of N_2 is in the form of

$$S_o = R/P(0) \quad (14)$$

where $P(0)$ is the DC gain of $P(S)$, if it exists. The value of S_o represents the actuating limitation of real system and will eliminate undesirable transient behaviors. The inputs of the plant are in the form of

$$u(n+1) = G(n+1) - Y_o/\mathrm{P}(0) \quad (15)$$

for the negative swing control with positive values of β, $G(j)$ and $F(j)$.

III. Numerical Examples

The first example [5] is shown in Fig.2, in which *P(S)* is in the form of

$$\mathrm{P}(s) = \frac{100}{s(1+0.1s)} \tag{16}$$

After it has been closed with feed-forward gain 0.03, the DC gain of the closed-loop subsystem *P(s)* is unity; i.e., *P(0)* is equal to one and the saturation level S_o described by Equation(14) is *R*. The sampling period T_S is selected to be equal to 1/40 second. The time responses of the overall system with the nonlinear digital controller for β=0.95 is shown in Fig.3. The magnitudes of reference inputs between 0 and 2 seconds are equal to 1; between 2 and 6 seconds are equal to -0.2, between 6 and 9 seconds are equal to 0.6, and between 9 and 12 seconds are equal to 1.2, in which gives output Y(solid-line), control input $G(j)$ (dash-line), and ratios $F(j)$ (dot-line) of $G(j)$. Fig.3 shows that all values of $G(j)$ and $F(j)$ are positive while the value of output *Y* tracking the negative value of the reference input R. The value of R may be positive or negative. Fig.3 shows also that ratios $F(j)$ are converged to be unities quickly; i.e., the controlled output tracks the reference input quickly. Fig.3 shows time responses for β=0.95 and sampling frequency equal to 60, 40, 30, 20Hz; respectively. Fig.4 shows that 40Hz (i.e., T_S =25ms) is fast enough for the considered system. From Figs. 3 and 4, one can see that undershot are worse than overshot. This is resulted from the asymmetric properties of Equation (9) which is shown in Fig.1. It is worth while to find a ratio function with symmetric properties.

Now, consider an electro-hydraulic velocity servo system [6] shown in Fig. 5 with system parameters given below:

$$K_S = 2.3\times 10^{-7}\sqrt{P_S - sign(X_V)P_L}\quad m^2/s$$

$$P_S = 1.4\times 10^{7}\ N_t/m^2\ ;\ \beta_o = 3.5\times 10^{7}\ N_t/m^2$$

$$V_t = 3.3\times 10^{-5}\quad m^2/rad$$

$$C_{tp} = 2.3\times 10^{-11}\ m^5/s/N_t$$

$$D_m = 1.6\times 10^{-5}\quad m^3/rad$$

$$J = 5.8\times 10^{-3}\quad Kg\text{-}m\text{-}s^2$$

$$B_m = 0.864\quad Kg\cdot m\cdot s/rad$$

$$K_v = 0.5\ m/v$$

The objective of the control is to keep the velocity ω_c of the hydraulic system following the desired reference input. The relation between the valve displacement X_V and the load flow rate Q_L is governed by the well known orifice law [7]

$$\begin{aligned} Q_L &= X_V\ K_j\sqrt{P_S - sign(X_V)P_L} \\ &= X_V\ K_S \end{aligned} \tag{17}$$

where K_j is a constant for specific hydraulic motor; *Ps* is the supply pressure; P_L is the load pressure and; K_s is the valve flow gain which varies at different operating points. The following continuity property of the servo valve and motor chamber yields

$$Q_L = D_m\,\omega_c + C_{tp}P_L + (V_t\text{-}4\beta_o)\dot{P}_L \tag{18}$$

where D_m is the volumetric displacement; C_{tp} is the total leakage coefficient; V_t is the total volume of the oil; β_o is the bulk modulus of the oil; and ω_c is the velocity of the motor shaft. The torque balance equation for the motor is in the form of

$$D_m P_L = J\,\dot{\omega}_c + B_m\,\omega_c + T_L \tag{19}$$

where B_m is the viscous damping coefficient and T_L is the external load disturbance which is assumed to be dependent upon the velocity of the shaft or slowly time varying as described by the following equation:

$$T_L = 20|\omega_c| \tag{20}$$

The step responses of the Example 2 for β=0.7, S_o =0.108, T_S =1/200 second, and the values of the reference inputs *R* between 0 and 0.3 seconds are equal to 1; between 0.3 and 0.6 seconds are equal to 0.4, between 0.6 and 0.9 seconds are equal to 0.8, and between 0.9 and 1.2 seconds are equal to 0.2, are shown in Fig. 6, in which gives the output

Y(solid-line), the series *G*(dash-line) and ratios *F*(dot-line) of the series. Fig.6 shows that the ratios $F(j)$ are converged to be unities quickly also.

IV. Conclusions

In this literature, a new nonlinear digital controller has been proposed for analyses and designs of sampled-data feedback control systems. The convergence of ratios was illustrated by two servo system examples. From simulation results, it can be seen that the nonlinear digital controller provided another possible control scheme for considered sampled-data feedback control systems and it is worthwhile to find symmetric $F(j)$ to get same overshot and undershot.

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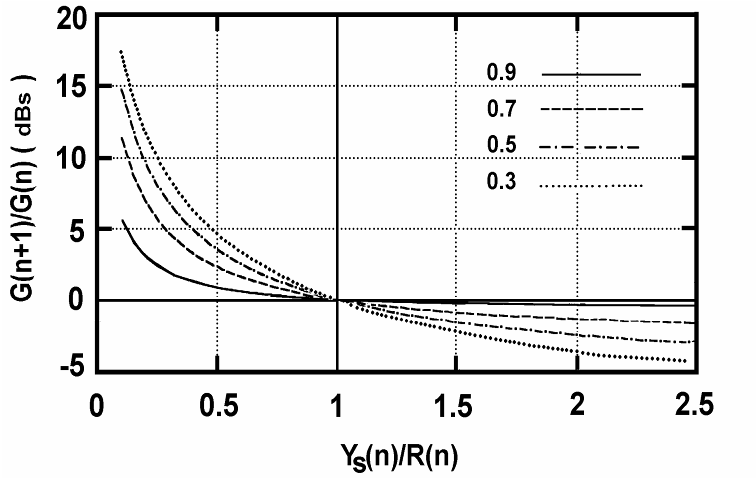

Fig.1 G(n+1)/G(n) Versus Y_S/R for β=0.9,0.7,0.5, and 0.3.

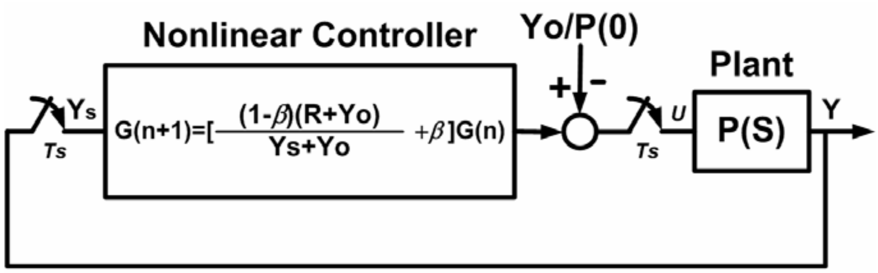

Fig.2 A Nonlinear Digital Controller.

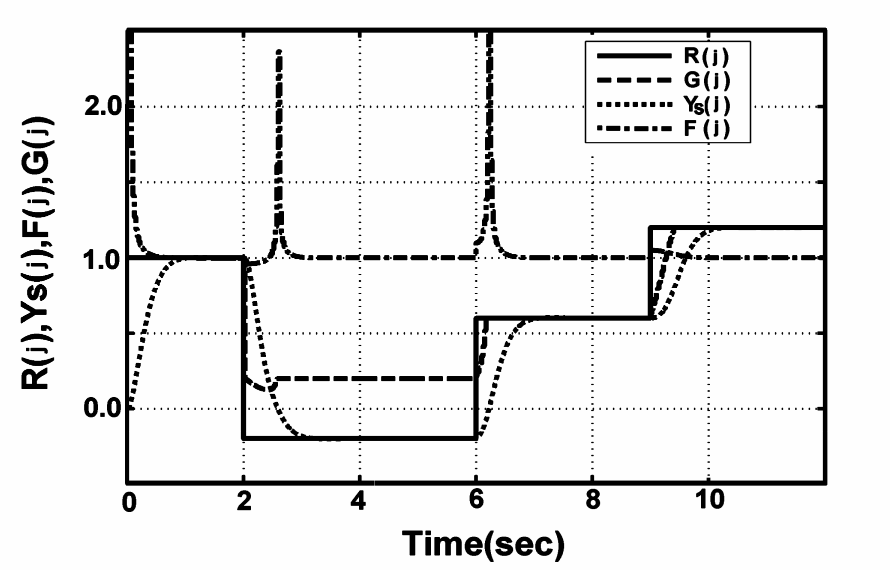

Fig.3 Time Responses of Example 1 for β=0.95.

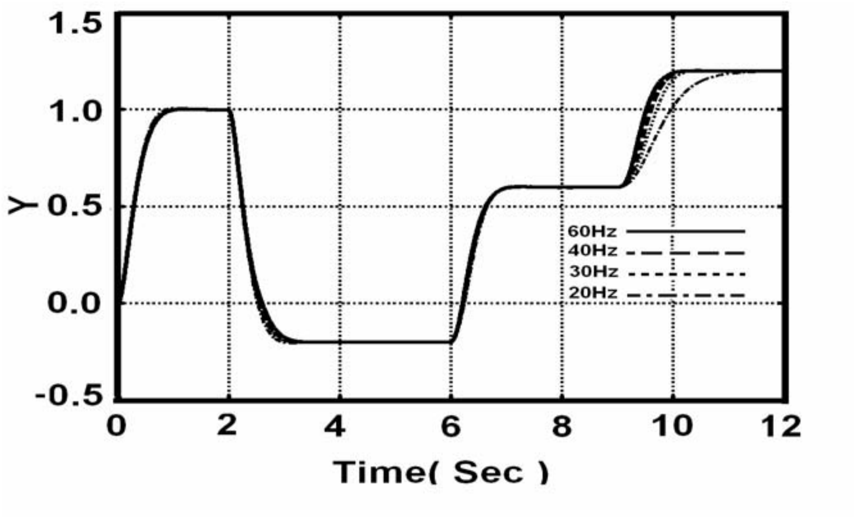

Fig.4 Time Responses of Example 1 for *Sampling Frequency* Equaling to 60, 40, 30, and 20Hz.

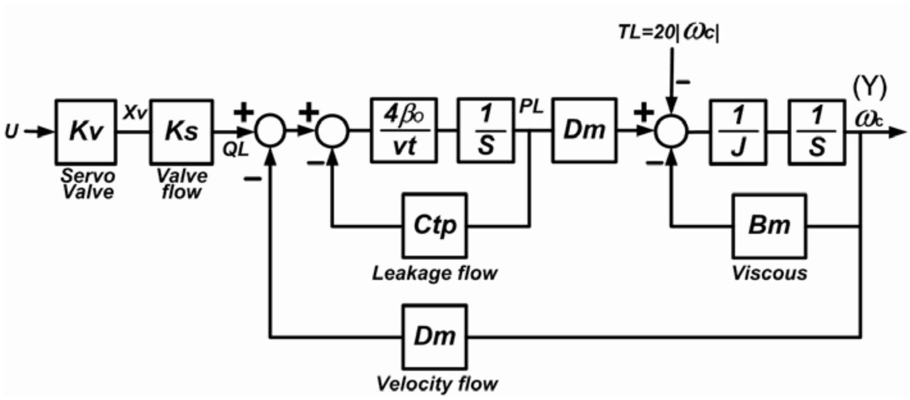

Fig.5 Mathematical Model of Example 2.

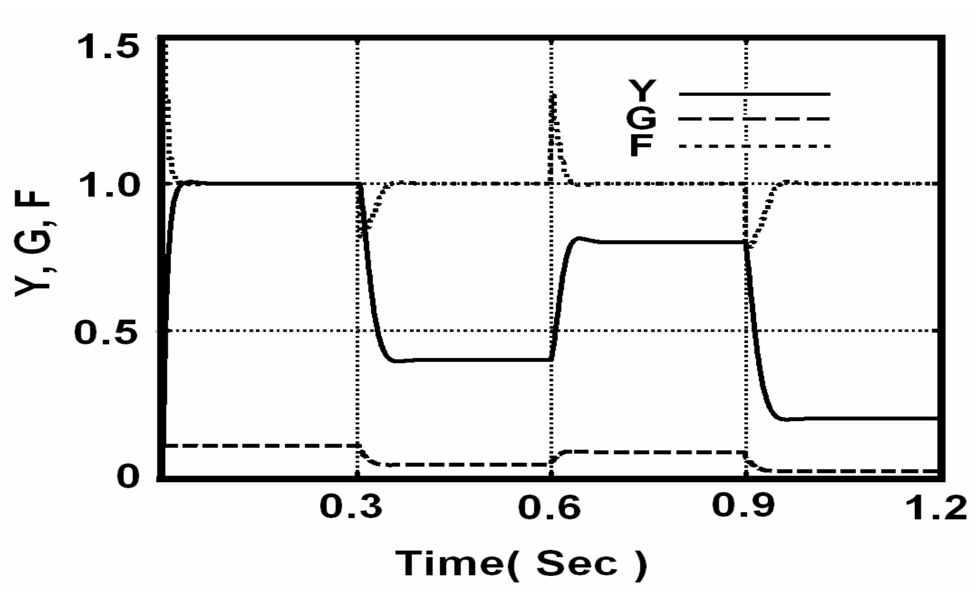

Fig.6 Time Responses of Example 2 for β=0.7.

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取樣回授控制系統之自動調整序列控制器

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摘　要

本文提出一個新的取樣回授控制系統的自動調整序列控制器，此一控制器爲應用數列收斂特性的非線性數位控制器。當利用受控體的輸出及參考輸入命令爲參數的數列之前後筆資料比收斂爲一時，代表受控體的輸出已跟隨參考輸入命令，完成控制目的。兩個伺服控制系統例子，被用來顯示此一控制器的優點。

關鍵詞:自動調整序列控制器，非線性控制，回授控制系統。

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