A Variable Structure System Controller Better Robustness

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Abstract

A variable structure system controller (VSSC) is proposed for robustness improvement of feedback control systems. The concept of the VSSC is illustrated in time domain, frequency domain and put into an analytical representation for analysis and design. It is applied to a simple position servo system for illustration and a complicated electro-hydraulic velocity servo system. Numerical simulating results show that the proposed VSSC improves the robustness while keeping performance of the considered systems significantly.

Keywords: Variable structure system controller, high robustness, servo control system.

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I. Introduction

Variable Structure System Control(VSSC) is characterized by a suite of feedback control laws and a decision rule [1-8]. The decision rule, called the switching function, has as its input some measure of the current system behavior and produces as an output the particular feedback controller that should be used at that instant of time. VSSC allows the design of high performance control systems that can be reliable, cheaply and easily implemented. For unity feedback control system, system responses can be partitioned into faster and slower regions to be switched. The command tracking error can be used to formulate the switching function.

The step responses are usually used to check the performance of the controlled system. They can be roughly partitioned into two regions. One is the transient region, and another is the steady-state region. They represent faster and slower responses; respectively. In general, high gain for fast response and phase lead for low overshot are required for good performance in the transient region, and low gain with tracking error integration is required for zero steady-state error in the steady-state region. The integrator will introduce the phase lag and degrade the performance, if it is used in the transient region. Variable structure system controllers are usually used for accomplishing two objects simultaneously. But the time(or check- point) for switching the variable structure controllers are ambiguous, especially for input-dependent nonlinear feedback control systems. It is expected to find an evident checkpoint for switching.

Consider a sinusoidal response shown in Fig.1. It can be partitioned into faster and slower regions according with the curvature of the waveform. The zero-crossing points (i.e.,π, 2π, 3π,…) with phase lead(ϕ_D) or phase lag(ϕ_D) can be used for partitioning. For instance, the region between π-ϕ_D and π+ϕ_D is defined as the faster region; and the region between π+ϕ_D and 2π-ϕ_D is defined as the slower region. The phase is used to replace the time for partitioning. The maximal value of ϕ_D is less than 90°. From Fig.1, one can see that the checkpoint for partitioning becomes evident and ϕ_D can be obtain by phase lead/lag elements. The partition becomes input amplitude independent.

The dash-line shown in Fig.1 between nπ-ϕ_D and nπ represents the inverse of the original waveform. It enlarges the decreasing inclination of the original waveform. It implies that phase lead is obtained by inversing the original waveform. Regions between $n\pi$ and $n\pi+\phi_D$ can not use the inversing property to get the lead phase. It gives lag phase. A variable structure system controller based upon this concept was first discussed by Foster, Giesekibg and Waymeyer [9], and modified by Buja and Souliaev [10] with the integration for zero steady state error. In this literature, significant improvement is applied by further enlarge the magnitude to enlarge the decreasing inclination. They imply that there are two adjustable parameters to the variable structure system control; i.e., phase lead for faster/slower partition and amplification at the signal inverse for phase lead compensation. The proposed VSSC is applied to a simple position servo system and a complicated electro-hydraulic velocity servo system. It will be seen that the proposed VSSC gets significant improvement for robustness.

II. The VSSC Controller

The configuration of the proposed VSSC is shown in Fig. 2, in which the part excludes the dash-lined macro block and N_3 and N_4 is the original VSSC[9]; and the part excludes N_4 is the original VSSC with the integration of error control signal X; the block $G_D(S)$ represents a lead element which provides a frequency-dependent leading phase ϕ_D for switching; and the block $G_I(S)$ represents the integrator to get high accuracy under external disturbance; N_1 and N_5 represent rectify(or absolute) blocks for outputs of error control signal and integrator; respectively; N_2 and N_3 represent the bang-bang blocks; i.e., zero crossing detectors; N_6 represents the switching control logic for the integrated error control signal y_2 added to y_1 or not; and parameter p of the N_4 represents the amplify gain at the signal inverse phase.

The operation concept of the VSSC is illustrated by introducing a sine wave to input X. The input/output signals are shown in Figs. 3(a) and 3(b). The signal X is the error control signal of the feedback control system; signal y_1 is the control signal output of the original VSSC [9]. The signal y_1 is derived from the nonlinear operations of N_1, $G_D(S)$ and N_2. The durations, which controlled by the leading phase ϕ_D of the compensator $G_D(S)$, between $n\pi-\phi_D$ and $n\pi$ is the so called "signal inverse phase". Fig. 3(a) shows that signs of another parts (i.e., non-inverse phase) of the signal y_1 are unaffected. The desired performance of the controlled system can be obtained by adjusting durations of the signal inverse phase [9, 10]; i.e., adjusting ϕ_D. Consider a first order lead compensator given below:

$$G_D(S) = \frac{ST_H + 1}{Sb_DT_H + 1} \qquad b_D < 1 \qquad (1)$$

and then the leading phase is found as in the form of

$$\phi_D = \tan^{-1}\left[\frac{\omega(1-b_D)T_H}{1+\omega^2 b_D T_H^2}\right] \quad 0 \le \phi_D \le \pi/2 \qquad (2)$$

where ω is the frequency in radian. Eq. (2) represents the VSSC is frequency dependent for ϕ_D is frequency dependent. The maximal value of ϕ_D is found as

$$\phi_m = \sin^{-1}\left(\frac{1-b_D}{1+b_D}\right) \qquad (3)$$

which occurs at

$$\omega_m = \frac{1}{\sqrt{b_D}T_H} \qquad (4)$$

The negative/positive value of multiplicative results of outputs of N_2 and N_3 represent the inverse/non-inverse phases of the VSSC; respectively. The value of parameter *p* in N_4 is the amplifying gain at the signal inverse phase shown in Fig.3. It will be seen that *p* becomes the major parameter rather than ϕ_D for the system performance. Note that the proposed VSSC will be returned to the original VSSC when the value of *p* is set to be unity.

The signal y_2 of Fig. 3(b) is the added integrated error control signal [2] with integrator

$$G_I(S) = 1/S \qquad (5)$$

and gain K_i. The addition of the signal y_2 is controlled by the operation of N_6. The signal ***Y*** represents the final control signal output of the proposed VSSC. The operation of the VSSC can be analyzed by means of the method of the describing function [11, 12]. The sine and cosine coefficients of the first harmonics of y_1 with sinusoidal error control signal $X = A\sin\alpha$ are

$$g_{e,o} = \frac{1}{\pi A}\int_0^{2\pi} y_1 \sin\alpha d\alpha$$

$$= K_p\left[\pi + (1+p')(-2\phi_D + \sin 2\phi_D)\right]/\pi \qquad (6a)$$

and

$$g_{o,o} = \frac{1}{\pi A}\int_0^{2\pi} y_1 \cos\alpha d\alpha$$

$$= K_p\left[(1+p')(1-\cos 2\phi_D)\right]/\pi \qquad (6b)$$

where *p'* is equal to *(p-1)/2*, and ϕ_D is defined by Eq.(2). Since ϕ_D is frequency dependent, the Eq. (6) is frequency dependent also. From Eq.(6), one can see that if the values of ϕ_D approaches to be zero, then the value of $g_{e,o}$ approaches to be K_P and $g_{e,o}$ approaches to be zero; i.e., the VSSC becomes to be linear gain K_P. This implies that the nonlinear operation occurs at high frequencies only.

The sine and cosine coefficients of the first harmonics of signal y_2 are

$$g_{e,i} = \frac{1}{\pi A}\int_0^{2\pi} y_2 \sin\alpha d\alpha$$

$$= K_i\left[3 + \cos 2\phi_D\right]/2\pi/\omega \qquad (7a)$$

and

$$g_{o,i} = \frac{1}{\pi A}\int_0^{2\pi} y_2 \cos\alpha d\alpha$$

$$= K_i\left[2\phi_D + \sin 2\phi_D\right]/2\pi/\omega \qquad (7b)$$

Then, the overall describing function is in the form of

$$g_e = g_{e,o} + g_{e,i} \qquad (8a)$$

and

$$g_o = g_{o,o} + g_{o,i} \qquad (8b)$$

Eq. (8) can be rewritten as

$$D_e = M_e \exp(j\phi_e) \qquad (9)$$

where

$$M_e = \sqrt{g_e^2 + g_o^2} \qquad (10a)$$

and

$$\phi_e = \tan^{-1}(g_o/g_e) \qquad (10b)$$

The frequency responses of Eq.(10) are shown in Fig.4 for T_H =0.20, b_D =0.025, K_P =0.5, K_i=0.05, and *p*=1, 1.5, 2.0. Fig.4 shows that the proposed VSSC has following properties:

1) gain and phase are separated; i.e., almost independent. This property is contrary to that of the linear controller;

2) no phase lag occurs with infinite gain at zero frequency. This property is contrast to that of the linear PI controller;
3) the larger value of *p*, the larger phase lead obtained with little perturbation of the gain versus frequency relationship.
Now, the controlled system with the VSSC will give fast response, low overshoot, and good robustness. This statement is based upon Eq. (10) and will be verified by digital simulations with following examples. Note that the value of p is limited with T_H =0.20, b_D =0.025, K_P =0.5, K_i =0.05. For instance, Fig.4 shows that the gain and phase become closed related after the value of p is greater than 1.5; i.e., it becomes the conventional lead compensation.

III. Applications

Consider a position servo system [13] with transfer function:

$$G_A(S) = \frac{K_{PA}}{S(1+T_A S)} \tag{11}$$

where K_{PA} =100 and T_A=0.2s. The time responses of the system with the proposed VSSC for T_H =0.20, b_D =0.025, K_P =0.5, K_i =0.05, and *p*=1.5 are shown in Fig. 5; in which solid-line shows plant output C, and dash-line shows input X of the VSSC, light-solid line shows the output Y of the VSSC. The bang-bang blocks N_2 and N_3 shown in Fig.2 are replaced by

$$N_2(X_1) = \frac{X_1}{|X_1| + 0.1} \tag{12}$$

and

$$N_3(X) = \frac{X}{|X| + 0.1} \tag{13}$$

respectively for suppressing the chattering of Y[6].

Fig.6 shows the simulating results for *p*=1, 1.5 and 2; respectively, in which gives responses of the controlled systems are almost the same. Fig.7 shows simulating results with maximal pure time delays T_D in control loop before system diverged for *p*=1, 1.5 and 2; respectively. The maximal time delay is corresponding to phase margin (*PM*).The relationship between them is in the form of

$$\exp(-T_D S) = \exp(-j2\pi NFT_D) \tag{14a}$$

and

$$PM = 360\ NF T_D ; \tag{14b}$$

where *NF* represents the fundamental oscillation frequency in Hertz. It is gain crossover frequency. Fig.7 gives maximal pure time delays before diverge. They are 76.5ms, 88.0ms and 94.5ms for *p*=1.0, 1.5 and 2.0; respectively. The corresponding phase margins are 55.1°, 63.4°, and 85.1°. It implies the proposed VSSC controller gives larger phase margin than that of the original VSSC, and the larger value of *p*, the larger phase margin will be. From Fig.6, it can be seen also that the variations of *p* does not disturb the performance found for *p*=1.

Now, consider an electro-hydraulic velocity servo system[14,15] shown in Fig. 8 with system parameters given in Table 1.The objective of the control is to keep the velocity ω_c of the hydraulic system following the desired reference input. The relation between the valve displacement X_V and the load flow rate QL is governed by the well-known orifice law [15]

$$QL = X_V\ K_j\ \sqrt{Ps - sign(X_V)PL}$$

$$= X_V\ K_S \tag{15}$$

where K_j is a constant for specific hydraulic motor; *Ps* is the supply pressure; *PL* is the load pressure and; K_S is the valve flow gain which varies at different operating points. The following continuity property of the servo valve and motor chamber yields

$$QL = D_m\ \omega_c + C_{tp}\ PL + (V_t \text{-}4\beta_o)\dot{PL} \tag{16}$$

where D_m is the volumetric displacement; C_{tp} is the total leakage coefficient; V_t is the total volume of the oil; β_o is the bulk modulus of the oil; and we is the velocity of the motor shaft. The torque balance equation for the motor is in the form of

$$D_m\ PL = J\dot{\omega}_c + B_m\ \omega_c + TL \tag{17}$$

where B_m is the viscous damping coefficient and *TL* is the external load disturbance which is assumed to be dependent upon the velocity of the shaft or slowly time varying as described by *TL* =20| ω_c |. The considered plant is a nonlinear system for the load flow rate QL is a nonlinear function of the valve displacement X_V and the load pressure PL.

The time responses of the controlled system are shown in Fig.9 for T_H =0.04, b_D =0.025, K_P =0.01, K_i =5.00, and p=1.0, 1.75; respectively. Fig.10 shows simulating results with maximal pure time delays T_D =44.2ms and 57.92ms for p=1 and 1.75; respectively. Fig.10 is used to show the system before diverged. The found gain crossover frequencies (NFs) are 6.48Hz and 6.71Hz; and phase margins (PMs) are 102.99° and 139.96°. That is the proposed controller gain 36.97° while keeping time response almost the same. Note the gain crossover frequencies are kept almost constant. From time responses and found phase margins, one can see that the robustness of the controlled system with the proposed VSSC is improved significantly.

IV. Conclusions

In this literature, a new variable structure system controller has been proposed and applied for analyses and designs of servo feedback control systems. The concept of the VSSC was illustrated in time domain and frequency domain and put into an analytical representation. Numerical simulating results with pure time delay robustness testing have been shown that the proposed VSSC gained 30° and 36.97° phase margins for Example 1 and Example 2; respectively while keeping original time responses almost the same.

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Table 1. System Parameters of Example 2.

K_s	2.3×10^{-7} $\sqrt{Ps - sign(X_V)PL}$	(m^2/s)
Ps	1.4×10^{7}	(N_t/m^2)
β_o	3.5×10^{7}	(N_t/m^2)
V_t	3.3×10^{-5}	(m^2/rad)
C_{tp}	2.3×10^{-11}	$(m^5/s/N_t)$
D_m	1.6×10^{-5}	(m^3/rad)
J	5.8×10^{-3}	$(Kg\text{-}m\text{-}s^2)$
B_m	0.864	(Kg-m-s/rad)
K_V	0.5	(m/v)
TL	$20\lvert\omega_c\rvert$	Kg-m

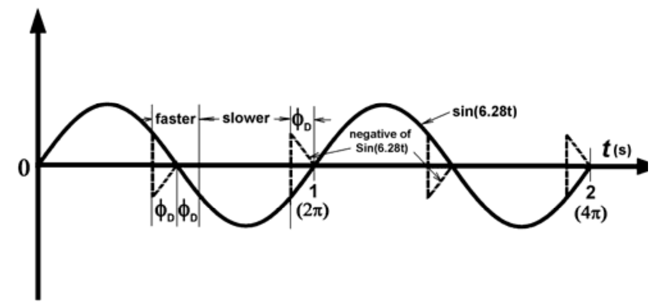

Fig.1 Faster and Slower Sections of a Sine Wave

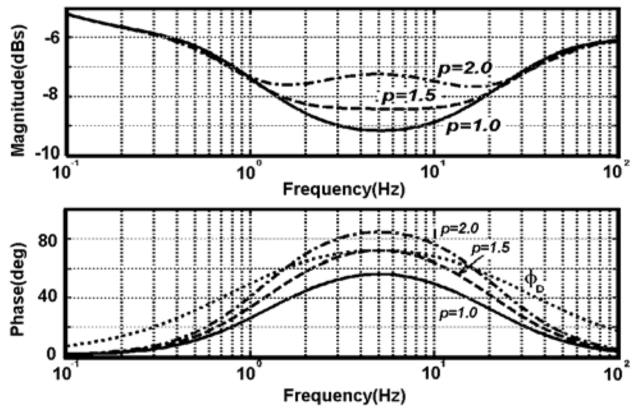

Fig.4 Frequency Responses of the VSSC Controller

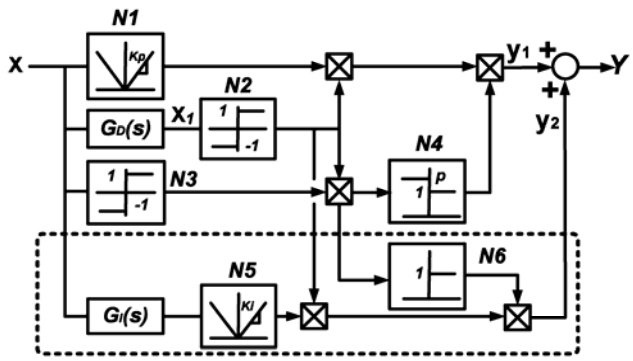

Fig.2 The Proposed VSSC Controller

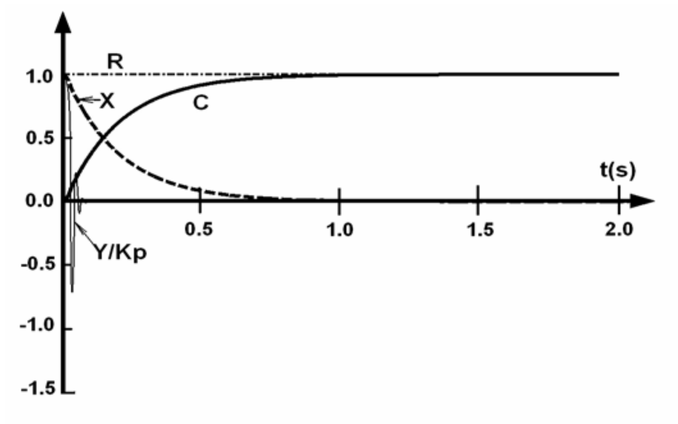

Fig.5 Time Responses of Example 1 with Bang-Bang Smoothing

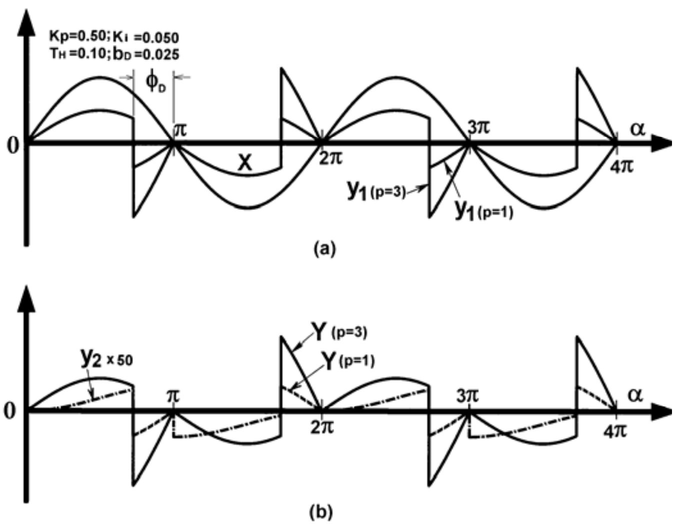

Fig.3 Time Responses of the VSSC Controller with Sine Wave X

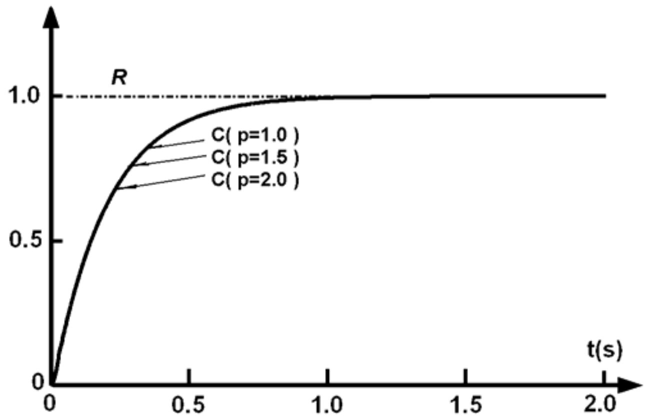

Fig.6 Time Responses of Example 1 for p=1, 1.5 and 2.0

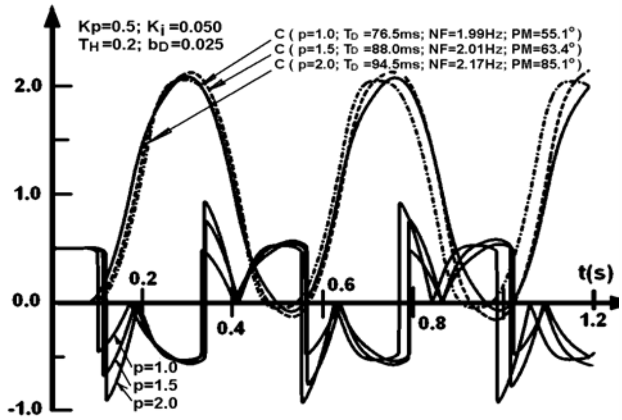

Fig.7 Maximal Pure Time Delay Simulation for p=1, 1.5 and 2.0

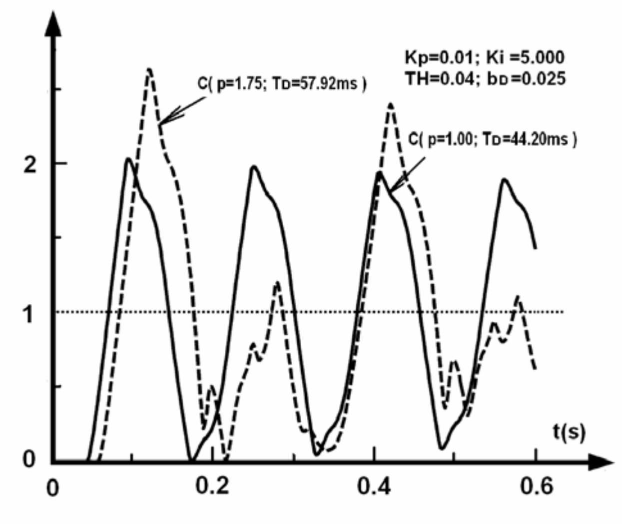

Fig.10 Maximal Pure Time Delay Simulation for p=1.0 and 1.75

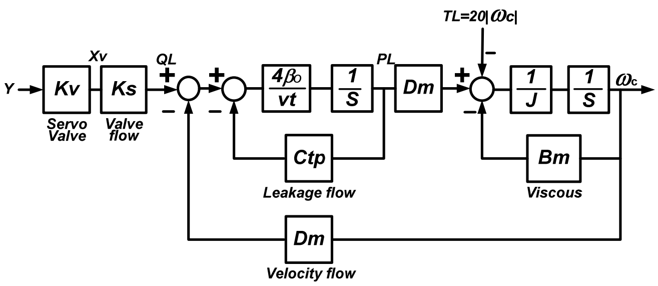

Fig.8 Block Diagram of Example 2

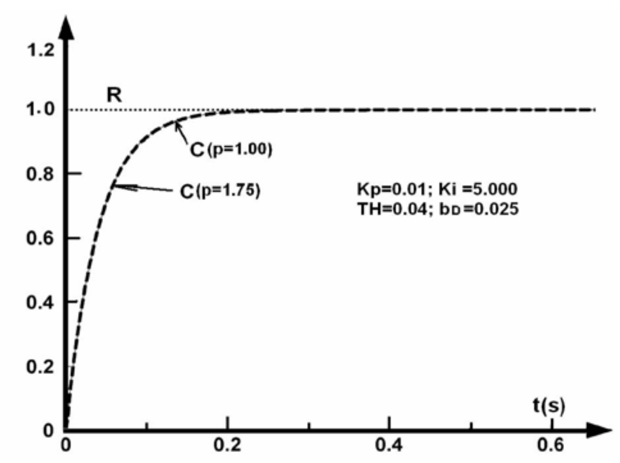

Fig.9 Time Responses of Example 2 for p=1.0 and 1.75

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高強健性可變系統架構控制器

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摘　要

本文提出一個可變系統架構控制器，並應用在伺服控制系統上，此一可變系統架構控制器可以在時域及頻域上分析，也可以用可解析的方程式表示，由兩個應用例子的模擬結果可知，所提的控制器可以在保持原有的性能下大幅增加強健性。

關鍵詞: 可變系統架構控制器，伺服控制系統，強健性。

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