Stable adaptive control for multivariable nonlinear system via Takagi-Sugeno fuzzy model

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Abstract

This paper presents a stable adaptive control methodology for a class of nonlinear systems. By Lyapunov's linearization method, the nonlinear system is first linearized on some operating points to produce linear dynamic models locally. Then, the Takagi-Sugeno fuzzy model (T-S fuzzy model) is adopted to aggregate these local models and formulates the approximation system. On the assumptions about the system's properties, the fuzzy system can be viewed as a linear perturbed system. The adaptive sliding mode controller is derived to ensure the asymptotic stability of the control system. In contrast to parallel distributed compensation, the proposed method is simple and easy for practical application. Two simulation cases, inverted pendulum system and articulated two-link robot, are employed to demonstrate the effectiveness of the proposed approach on stabilization and tracking control.

Key Words: T-S fuzzy model、 Adaptive sliding mode control.

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I. Introduction

In industry, most control systems are nonlinear and too complex to have their exact mathematic models. Mathematically these systems can be represented by a nominal model (linear or nonlinear) with uncertainty (structured or unstructured). The analysis and synthesis of those control systems consider not only stabilization problems but also the robustness in the presence of uncertainties. Over the past decades, the issue has drawn much research interest and many significant developments have been reported. Among various kinds of control studies, the fuzzy control is undoubtedly regarded as one of the most active and fruitful disciplines.

The Takagi-Sugeno (T-S) fuzzy system, first introduced in[1], is known as a universal approximator for nonlinear system or function. Essentially it is an interpolation method. The physical nonlinear systems are assumed to be approximated by a set of linear or nonlinear models around some local operating points. These local models are then smoothly aggregated via the fuzzy inferences. Therefore, the T-S fuzzy model provides a compact and flexible mathematic description for complex or ill-defined system. Based on this idea, many nonlinear control approaches have been developed.

Initially, sufficient condition for stabilization control deduced in[2] was established on the hypothesis that there must is a common matrix P for each fuzzy local systems. However, the common matrix P was not easy to obtain until Tanaka[3] introduced linear matrix inequality (LMI) method to solve this problem. In fact, the LMI is a very powerful tool for the analysis and design of control systems[4]. Most of the published results are based on LMI approach to derive sufficient conditions for the stabilization of fuzzy systems. However, the existence of the solution is not guaranteed. As the number of fuzzy rules increases or too many constraints are imposed, the solution could be infeasible[5].

In contrast to LMI's method, the adaptive control method is considered as an alternative to dealing with the control of fuzzy systems. In particular, based on the universal approximation theorem and by incorporating fuzzy system into adaptive control scheme, the adaptive fuzzy control approaches are presented in[6, 7, 8]. Moreover, an adaptive fuzzy based controller combined with sliding mode control has been studied in[9, 10], where the controlled system is nonlinear and in controller companion form.

In this paper, we propose a conventional control approach for nonlinear multivariable systems. The controlled system does not require being in controller companion form. When some assumptions regarding the properties of the system hold, the fuzzy system can be transformed into the conventional sliding mode control scheme. With the help of fuzzy basis function and adaptive mechanism, the system uncertainties and the corresponding upper bounds can be estimated. Based on the Lyapunov functional analysis, the adaptive laws are constructed to guarantee the stability of the control system. Finally, numerical examples are presented to verify the effectiveness of the proposed control method.

II. Problem statements

Consider a nonlinear system described by

$$\dot{x}(t) = f(x(t),\ u(t)) \qquad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^p$ is the control input vector. It is assumed that $f(x(t), u(t))$ is continuously differentiable with respect to $x(t)$ and $u(t)$ and has linear dynamics around some operating points (x_i, u_i). Then, by using a linearization method[11], we have

$$\dot{x}(t) = A_i x(t) + B_i u(t) + d_i(t),\ i = 1,2,\cdots,q \quad (2)$$

where $A_i = (\partial f / \partial x)_{x=x_i}$, $B_i = (\partial f / \partial u)_{u=u_i}$, and $d_i(t)$ stands for the approximation error. Since the linearized models depict system dynamics in local region around the operating points, it is advisable to aggregate these models and formulate the approximation system by T-S fuzzy inference. With proper selection and definition of input variables and membership functions, the T-S fuzzy inferences are in the form of

$$\begin{aligned} R^l &: \text{If } z_1(t) \text{ is } M_1^l \text{ and} \cdots \text{and } z_j(t) \text{ is } M_j^l \\ &\quad \text{then } \dot{x}(t) = A_l x(t) + B_l u(t) + d_l(t) \\ &\quad l = 1,\ 2,\ \cdots,\ q\ . \end{aligned} \quad (3)$$

where M_k^l is the fuzzy set $(k = 1,2,\cdots,j)$ and $z(t) = [z_1(t), z_2(t), \cdots, z_j(t)]^T$ is the premise variable vector associated with the system states and inputs.

By center of gravity defuzzification, the output of fuzzy system is inferred as

$$\dot{x}(t) = \frac{\sum_{l=1}^{q} w_l(z)[A_l x(t) + B_l u(t) + d_l(t)]}{\sum_{l=1}^{q} w_l(z)} \quad (4)$$

where $w_l(z) = \prod_{i=1}^{j} M_i^l(z_i)$ and $M_i^l(z_i)$ is the grade of membership function M_i^l corresponding $z_i(t)$. Let $\mu_l(z)$ be defined as

$$\mu_l(z) = \frac{w_l(z)}{\sum_{l=1}^{q} w_l(z)} \quad (5)$$

Then (4) becomes

$$\dot{x}(t) = \sum_{l=1}^{q} \mu_l(x)[A_l x(t) + B_l u(t) + d_l(t)] \quad (6)$$

It is obviously $\sum_{l=1}^{q} \mu_l(z) = 1$ and $\mu_l(z) \ge 0$ for $l = 1,2,\cdots,q$.

Inspired by the works of [8], we modify the fuzzy system (6) as

$$\begin{aligned} \dot{x}(t) &= (A_o + \Delta A(t,x))x(t) \\ &\quad + (B_o + \Delta B(t,x))u(t) + D(t,x) \end{aligned} \quad (7)$$

where

$$\begin{aligned} &A_o = \frac{1}{q}\sum_{i=1}^{q} A_i,\ B_o = \frac{1}{q}\sum_{i=1}^{q} B_i, \\ &\Delta A(t,x) = \sum_{i=1}^{q} \mu_i(z)(A_i - A_o), \\ &\Delta B(t,x) = \sum_{i=1}^{q} \mu_i(z)(B_i - B_o),\ D(t,x) = \sum_{i=1}^{q} \mu_i(z) d_i\ . \end{aligned} \quad (8)$$

From (7), it can be seen that the T-S fuzzy system is depicted as a perturbed linear system with the nominal matrices (A_o, B_o), the perturbations $(\Delta A(t,x), \Delta B(t,x))$, and the modeling error or disturbance $D(t,x)$.

III. Design of adaptive sliding mode fuzzy control

To complete the derivation, we impose the following assumptions on system (7).

Assumption 1. The pair (A_o, B_o) is completely controllable.

Assumption 2. The state $x(t)$ is available for measurement.

Assumption 3. The perturbations *(ΔΔA(tx), ΔB(t, x))* and the modeling error *D(t, x)* are matched. That is, there exist matrices $E(\cdot): R \times R^n \to R^{p \times n}$, $F(\cdot): R \times R^n \to R^{p \times p}$, and $G(\cdot): R \times R^n \to R^{p \times 1}$, such that

$$\Delta A(t,x) = B_o E(t,x),$$

$$\Delta B(t,x) = B_o F(t,x),\ D(t,x) = B_o G(t,x)\ . \quad (9)$$

Based on the assumptions, the system (7) can be rewritten as

$$\dot{x}(t) = A_o x(t) + B_o(u(t) + \xi(t,x))\ . \quad (10)$$

where $\xi(t,x) \in R^p$ denotes the lumped uncertainty.

For the system with uncertainty, the sliding mode control (SMC)[11] is a useful control strategy. It provides a systematic approach to solve the problem of maintaining stability and consistent performance. The design of SMC consists of two phases. The first phase is to construct an appropriate sliding surface so that the system conducted on the sliding surface will produce a desired behavior. That is, the system is invariant to the uncertainty or disturbance and consistent performance is achieved while the states are maintained on the surface. The second phase is to design the control law so that the sliding condition[11] is satisfied. In particular, once on the surface, the system trajectories will remain there for all the subsequent time.

Since the uncertainty ξ is assumed to be matched, we define the time-varying sliding surface as

$$\Omega = \{x : S(x) = Cx = 0\} \tag{11}$$

where C is a $p \times n$ constant matrix such that CB_o is nonsingular and the reduced $(n-p)$ order equivalent system restricted to the surface is asymptotically stable. Consider the sliding condition

$$\mathrm{S}^{\mathrm{T}}\dot{\mathrm{S}} < -\mathrm{K}\|\mathrm{S}\|^2 \tag{12}$$

where K is a positive real number and $\|\cdot\|$ denote the Euclidean norm.

Differentiating $S(x)$ with respect to time gives

$$\begin{aligned} \dot{S}(x) &= C\dot{x}(t) \\ &= CA_o x(t) + CB_o(u(t) + \xi(t,x)) \end{aligned} \tag{13}$$

To satisfy the sliding condition, the control law is chosen as

$$u = -(CB_o)^{-1}(CA_o x + KS) - \xi \tag{14}$$

Since there is an uncertainty ξ, the design of controller requires estimation of the uncertainty. Generally, this can be completed by using an adaptive mechanism to evaluate the norm value of uncertainty. However, when the system is complex, it is difficult to construct the adaptive mechanism because the relationship between measurement variables and system uncertainty is ambiguous. In this situation, the fuzzy inference is useful for manipulating adaptive estimation.

The idea that the fuzzy system is a universal approximator that can approximate any real continuous function on a compact set to an arbitrary accuracy is well known. Many explored adaptive fuzzy control approaches are based on this concept [6-10]. In the following derivation, we will use the fuzzy basis function to approximate the uncertainty ξ and develop the adaptive laws for estimation of the uncertainty and the corresponding upper bounds.

Consider the following Mamdani type fuzzy inference that is to approximate the i^{th} element of ξ, ξ_i, as

$$\begin{aligned} &\mathrm{R}^{m}: \text{If } x_1(t) \text{ is } \widetilde{M}_1^m \text{ and} \cdots \text{and } x_n(t) \text{ is } \widetilde{M}_n^m \\ &\quad \text{then } \xi_i \text{ is } \widetilde{D}_{im},\ m = 1, 2, \cdots, r\ . \end{aligned}$$

The output of fuzzy system is inferred by

$$\begin{aligned} \hat{\xi}_i(x \mid \theta_i) &= \frac{\sum_{m=1}^{r} \theta_{im} \left(\prod_{h=1}^{n} \mu_{\widetilde{M}_h^m}(x_h) \right)}{\sum_{m=1}^{r} \prod_{h=1}^{n} \mu_{\widetilde{M}_h^m}(x_h)} \\ &= \theta_i^T \omega(x) \end{aligned} \tag{15}$$

where $\theta_i = (\theta_{i1}, \theta_{i2}, \cdots, \theta_{ir})^T$ is an adjustable parameter vector, θ_{im} is the center of $\widetilde{D}_{im}$ for $i = 1,2,\cdots,p$, and $\omega(x)$ is called the fuzzy basis function. The estimation of ξ is given by $\hat{\xi}(x \mid \theta) = \theta^T \omega(x)$ and $\theta \in R^{r \times p}$. Define an optimal parameter matrix as

$$\theta^* = \arg \min_{\theta \in \Omega_\theta} \{ \sup_x \| \hat{\xi}(x \mid \theta) - \xi(t,x) \| \} \tag{16}$$

and assume

$$\left\| \hat{\xi}(x \mid \theta^*) - \xi(t,x) \right\| \le \varepsilon_1 + \varepsilon_2 \|x\| \tag{17}$$

where ε_1 and ε_2 are unknown positive constants and will be estimated via adaptive mechanism.

Choose the control law to be

$$\begin{aligned} u = &-(CB_o)^{-1}(CA_o x + KS) - \hat{\xi} \\ &- \frac{\hat{\varepsilon}_1 + \hat{\varepsilon}_2 \|x\|}{\left\| B_o^T C^T S \right\|} B_o^T C^T S \end{aligned} \tag{18}$$

where $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ are the estimations for ε_1 and ε_2 and $\|\cdot\|$ for the matrix denotes the induced norm. Substituting (18) into (13) gives

$$\begin{aligned} \dot{S} &= -KS - CB_o(\xi - \hat{\xi}) - \frac{CB_o(\hat{\varepsilon}_1 + \hat{\varepsilon}_2 \|x\|)}{\left\| B_o^T C^T S \right\|} B_o^T C^T S \\ &= -KS + CB_o \tilde{\xi} + CB_o(\xi - \hat{\xi}^*) \\ &\quad - \frac{CB_o(\varepsilon_1 + \varepsilon_2 \|x\|)}{\left\| B_o^T C^T S \right\|} B_o^T C^T S + \frac{CB_o(\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2 \|x\|)}{\left\| B_o^T C^T S \right\|} B_o^T C^T S \end{aligned} \tag{19}$$

where

$$\tilde{\xi} = \hat{\xi}(x \mid \theta^*) - \hat{\xi}(x \mid \theta), \quad \hat{\xi}^* = \hat{\xi}(x \mid \theta^*),$$

$\tilde{\varepsilon}_1 = \varepsilon_1 - \hat{\varepsilon}_1$ and $\tilde{\varepsilon}_2 = \varepsilon_2 - \hat{\varepsilon}_2$. Multiplying S^T to the right side of (21) gives

$$\begin{aligned} S^T \dot{S} = &-K\|S\|^2 + S^T CB_o \tilde{\xi} + S^T CB_o(\xi - \hat{\xi}^*) \\ &-(\varepsilon_1 + \varepsilon_2 \|x\|)\left\| B_o^T C^T S \right\| + (\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2 \|x\|)\left\| B_o^T C^T S \right\| \end{aligned} \tag{20}$$

Define the parameter error as $\tilde{\theta} = \theta^* - \theta$ and choose the Lyapunov functional candidate

$$\begin{aligned} V(t) = &\frac{1}{2} S^T S + \frac{1}{2\eta_\theta} tr[\tilde{\theta}^T \tilde{\theta}] \\ &+ \frac{1}{2\eta_1} \tilde{\varepsilon}_1^2 + \frac{1}{2\eta_2} \tilde{\varepsilon}_2^2 \end{aligned} \tag{21}$$

where η_θ, η_1 and η_2 are the adaptation rates. The time derivative of *V(t)* along the dynamics of *S* is

$$\begin{aligned} \dot{V}(t) &= S^T \dot{S} - \frac{1}{\eta_\theta} tr[\tilde{\theta}^T \dot{\theta}] - \frac{1}{\eta_1} \tilde{\varepsilon}_1 \dot{\hat{\varepsilon}}_1 - \frac{1}{\eta_2} \tilde{\varepsilon}_2 \dot{\hat{\varepsilon}}_2 \\ &= -K\|S\|^2 + S^T CB_o \tilde{\xi} + S^T CB_o(\xi - \hat{\xi}^*) \\ &\quad - (\varepsilon_1 + \varepsilon_2 \|x\|)\left\| B_o^T C^T S \right\| + (\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2 \|x\|)\left\| B_o^T C^T S \right\| \\ &\quad - \frac{1}{\eta_\theta} tr[\tilde{\theta}^T \dot{\theta}] - \frac{1}{\eta_1} \tilde{\varepsilon}_1 \dot{\hat{\varepsilon}}_1 - \frac{1}{\eta_2} \tilde{\varepsilon}_2 \dot{\hat{\varepsilon}}_2 \\ &\le -K\|S\|^2 + \frac{1}{\eta_\theta} tr[\tilde{\theta}^T (\eta_\theta \omega S^T CB_o - \dot{\theta})] \\ &\quad + \left\| \xi - \hat{\xi}^* \right\| \left\| B_o^T C^T S \right\| - (\varepsilon_1 + \varepsilon_2 \|x\|)\left\| B_o^T C^T S \right\| \\ &\quad + \frac{1}{\eta_1} \tilde{\varepsilon}_1 (\eta_1 \left\| B_o^T C^T S \right\| - \dot{\hat{\varepsilon}}_1) \\ &\quad + \frac{1}{\eta_2} \tilde{\varepsilon}_2 (\eta_2 \left\| B_o^T C^T S \right\| \|x\| - \dot{\hat{\varepsilon}}_2 \end{aligned} \tag{22}$$

Notably, $\tilde{\xi} = \theta^{*T} \omega - \theta^T \omega = \tilde{\theta}^T \omega$. If the following adaptive laws are employed

$$\begin{aligned} \dot{\theta} &= \eta_\theta \omega S^T CB_o, \\ \dot{\hat{\varepsilon}}_1 &= \eta_1 \left\| B_o^T C^T S \right\|, \\ \dot{\hat{\varepsilon}}_2 &= \eta_2 \left\| B_o^T C^T S \right\| \|x\| . \end{aligned} \tag{23}$$

then we have $\dot{V}(t) \le -K\|S\|^2$ (24)

Consequently, *V(t)* is a bounded function, which implies that S, $\tilde{\theta}$, $\hat{\varepsilon}_1$, and $\hat{\varepsilon}_2$ are all bounded. From the definition of *S* one can conclude that the state *x* will be confined in certain range. To prove $\lim_{t \to \infty} S(t) = \mathbf{0}$, we define $V_1(t)$ as

$$V_1(t) = V(t) - \int_0^t (\dot{V}(\tau) + K\|S(\tau)\|^2) d\tau \tag{25}$$

It follows that $V_1(t) \ge 0$. Differentiating $V_1(t)$ with respect to time gives

$$\begin{aligned} \dot{V}_1(t) &= \dot{V}(t) - (\dot{V}(t) + K\|S(t)\|^2) \\ &= -K\|S(t)\|^2 \end{aligned} \tag{26}$$

Moreover, $\ddot{V}_1(t) = -2KS^T \dot{S}$ is bounded. By using the Barbalat's lemma[11], we have $\lim_{t \to \infty} S(t) = 0.$

That implies $\lim_{t \to \infty} x(t) = 0$ and the closed-loop system is asymptotically stable.

Based on the above discussion, we may briefly summarize the design procedure for the controlling nonlinear multivariable systems.

Step 1: Select certain operating points concerning with the system performance and perform linearization on these points to obtain the local linear models.

Step 2: Choose appropriate linguistic variables and define the corresponding membership functions to build the T-S fuzzy model.

Step 3: Calculate the nominal matrices (A_o, B_o) and examine the controllability.

Step 4: Verify the matching condition(9).

Step 5: Construct an appropriate sliding surface in which the reduced-order equivalent system will be asymptotically stable on this surface.

Step 6: Apply the control law(18) and adjust the parameters θ, $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ by adaptive law(23).

IV. Simulation

In this section we take two examples to illustrate the design procedure and verify the effectiveness of the proposed algorithm.

Example 1. The control objective is focused on balancing an inverted pendulum on a cart. The dynamic equations of the pendulum are given by

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)}$$

where x_1 is the angle of the pendulum from the equilibrium position, x_2 is the angular velocity, and u is the force applied to the cart. The parameters are given as follows: $g = 9.8m/s^2$ the gravity constant, $m = 2.0kg$ the mass of the pendulum, $M = 8kg$ the mass of the cart, $2l = 1.0m$ the length of the pendulum, and $a = 1/(m+M)$.

The nonlinear system has linear models on 0 and $\pm\frac{\pi}{2}$ and the T - S fuzzy system for approximation is defined as[12]:

R^1 : If x_1 is about 0, then $\dot{x} = A_1x + B_1u + d_1$,

R^2 : If x_1 is about $\pm\frac{\pi}{2}$, then $x = A_2x + B_2u + d_2$.

The matrices A_1, A_2, B_1, and B_2 are the local linear models as following:

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{4l/3 - aml} & 0 \end{bmatrix},\quad A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -\dfrac{a}{4l/3 - aml} \end{bmatrix},\quad B_2 = \begin{bmatrix} 0 \\ -\dfrac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix}.$$

where $\beta = \cos(88^\circ)$. Then, according to(10), we obtain the nominal model

$$A_o = \begin{bmatrix} 0 & 1 \\ 13.3271 & 0 \end{bmatrix},\ B_o = \begin{bmatrix} 0 \\ -0.0909 \end{bmatrix}.$$

Since the local linear models are in controllable canonical form and the uncertainties satisfy the matching condition, the adaptive sliding mode control can be applied. Define the sliding surface as $s = [\lambda\ 1]$. The adaptive fuzzy mechanism for estimating ξ is defined as

$$R^l\ :\ If\ \chi_1\ is\ \widetilde{M}_l,\ then\ \ \xi\ is\ \widetilde{D}^l,\ l = 1,\ 2,\ ...,\ 5\,.$$

The corresponding membership functions are plotted in Fig.1. The universe of discourse $[-\frac{\pi}{2}, \frac{\pi}{2}]$ is normalized over this range. Moreover, the proposed method is compared with the conventional parallel distributed compensation (PDC) with state feedback gains $K_1 = [1900.13\ \ 2483.11]$ and $K_2 = [3.33\ \ 7.12]$, which are calculated by LMI's method.

The initial value of θ is set to be 1e-3*[1 1 –1 –1 –1]^T, and the values of $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ are 0 and 1 respectively. The simulation results are plotted in Fig.2 and Fig.3, where solid line presents adaptive T-S control with $\lambda = 20$, dash line depicts adaptive T-S control with $\lambda = 10$, and dash-dot line stands for control by PDC. We choose two different values of λ to inspect the convergent rate of x_1.

The simulation reveals that the large λ produces fast convergence and both can achieve stable control.

Example 2. Consider a two-link articulated robot described by the following dynamic equations[11]:

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_2 - h\dot{q}_1 \\ h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

where

$$\begin{aligned} H_{11} &= a_1 + 2a_3 \cos q_2 + 2a_4 \sin q_2, \\ H_{12} &= H_{21} = a_2 + a_3 \cos q_2 + a_4 \sin q_2, \\ H_{22} &= a_2, \\ h &= a_3 \sin q_2 - a_4 \cos q_2 \ . \end{aligned}$$

and

$$\begin{aligned} a_1 &= I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2, \\ a_2 &= I_e + m_e l_{ce}^2, \\ a_3 &= m_e l_1 l_{ce} \cos\delta_e, \\ a_4 &= m_e l_1 l_{ce} \sin\delta_e. \end{aligned}$$

The values of parameter used in the simulation are as follows:

$$m_1 = 1,\ l_1 = 1,\ m_e = 2,\ \delta_e = 30^\circ,\ I_1 = 0.12,\ l_{c1} = 0.5,\ I_e = 0.25,\ I_{ce} = 0.6\ .$$

Since the inertia matrix H is uniformly positive definite, the system dynamics can be rewritten as

$$\begin{aligned} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} &= \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}^{-1}\begin{bmatrix} h\dot{q}_2 & h(\dot{q}_1 + \dot{q}_2) \\ -h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \\ &+ \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}^{-1}\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \\ &= F(q,\ \dot{q}) + G(q)\tau \end{aligned}$$

We use T-S fuzzy model to present the system dynamics and apply adaptive sliding mode control to obtain the desired performance.

First, the Coriolis torque term is linearized on certain operating points to build the linear model

$$H(q_2)\ddot{q} = A_i\dot{q} + \tau + d_i,\ i = 1,2,\cdots,m$$

where

$$A_i = \frac{\partial}{\partial\dot{q}}\left(\begin{bmatrix} h\dot{q}_2 & h(\dot{q}_1 + \dot{q}_2) \\ -h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}\right)\Bigg|_{\dot{q}=\dot{q}_i}$$

Then, the T-S fuzzy model is defined as

$$R^i : \text{If } q_2 \text{ is about } \theta_i^\circ \text{ and } \dot{q} \text{ is about } \dot{q}_i$$
$$\text{then } \ddot{q} = H^{-1}(q_2\ (\theta_i^\circ))\ [A_i\dot{q} + \tau + d_i]$$

for $i = 1,2,\cdots,k$. In simulation, we choose three operating points, ± 1 and 0, for θ_i° and $\dot{\mathbf{q}}_i$ is set near zero ($\dot{q}_1 = \dot{q}_2 =$ 1e-6). By some calculation, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -7.23\times10^{-8} & -6.99\times10^{-8} \\ 0 & 0 & 0.164\times10^{-6} & 7.47\times10^{-8} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.68 & -1.29 \\ -1.29 & 3.55 \end{bmatrix}\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \xi_1 \\ \xi_2 \end{bmatrix}$$

where $x_1 = q_1,\ x_2 = q_2,\ x_3 = \dot{q}_1,\ x_4 = \dot{q}_2,$ and ξ_1 and ξ_2 denote the uncertainties. The nominal system is controllable and the uncertainties satisfy matching condition.

The next step is to define the sliding surface. It is advisable to choose $S(x)$ as

$$S(x) = Cx = \begin{bmatrix} \lambda & 0 & 1 & 0 \\ 0 & \lambda & 0 & 1 \end{bmatrix} x$$

When the states are maintained on $S(x)$, the dynamics of the reduced-order equivalent system becomes

$$\begin{aligned} \dot{\chi}_1 &= \chi_3 = -\lambda\chi_1, \\ \dot{\chi}_2 &= \chi_4 = -\lambda\chi_4\ . \end{aligned}$$

Obviously, it is asymptotically stable.

The adaptive fuzzy mechanism for the estimation of ξ is given by

$$R^1: \text{If } q_2 \text{ is } \widetilde{M}_1, \text{ then } \xi_1 \text{ is } \widetilde{D}_1^l \text{ and } \xi_2 \text{ is } \widetilde{D}_2^l, \quad l = 1,\ 2,\ \cdots,\ 5\ .$$

The universe of discourse of q_2 and the corresponding membership functions are given in Fig.1 The initial values of parameters are chosen as

$$\theta = 3 * \begin{bmatrix} 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 \end{bmatrix}^T$$

$\hat{\varepsilon}_1 = \hat{\varepsilon}_2 = 5$, and λ =50.

We first investigate a step response of the robot motion. The robot is initially at rest, $q_1 = q_2 = 0$, and then a step-input command $(q_{d1} = 60^\circ,\ q_{d2} = 90^\circ)$ is issued. The performances of position control errors and the corresponding input torques are plotted in Fig.4 and Fig.5, where solid line presents control by our method and dash line stands for PD control. In the simulation, our approach is compared with PD control described in [11]. It is shown that the control results of both approaches are stable. The next case is to examine the tracking ability. The robot has to follow desired trajectories

$q_{d1} = 30^\circ(1-\cos(2\pi t)),\ \ q_{d2} = 45^\circ(1-\cos(2\pi t))$.

The performances of tracking errors and the control torques are plotted in Fig.6 and Fig.7, where solid line presents control by our method, dash line depicts conventional sliding mode control[11], and dash-dot line stands for adaptive control[13]. We choose two different values of λ to inspect the convergent rate of x_1. It can be seen that our method has the best performance.

Although the derivation is focused on the stabilization, it can easily be transformed into tracking control. In this situation, the sliding surface is modified as

$$S(\tilde{\chi}) = C\tilde{\chi},\ \ \tilde{\chi} = \chi - \chi_d$$

where x_d denotes the desired trajectory. Moreover, the control law (18) is replaced by

$$u = -(CB_o)^{-1}(CA_o\tilde{x} + KS + CA_o x_d - C\dot{x}_d) - \hat{\xi} - \frac{\hat{\varepsilon}_1 + \hat{\varepsilon}_2\|x\|}{\|B_o^T C^T S\|} B_o^T C^T S$$

V. Conclusion

This paper presents a systematic design approach for controlling multivariable nonlinear system based on T-S fuzzy model. The concept is simple and easy to apply. In contrast to PDC control scheme, the proposed controller is constructed without considering the linear matrix inequalities. Therefore, there is no common *P* problem. When some assumptions regarding the properties of the system hold, the sliding mode control can be applied such that the asymptotic stability of the global system is ensured. The effectiveness of proposed approach is illustrated by computer simulations of the inverted pendulum and the two-link robot.

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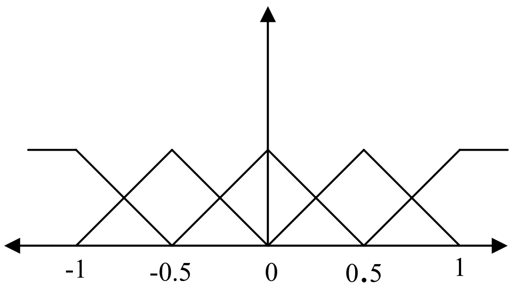

Fig.1 The membership functions

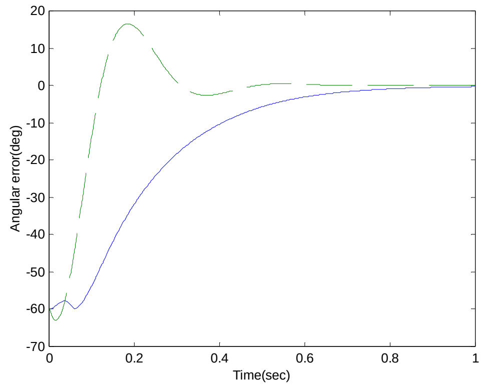

Fig.4 (a) Position error of link 1

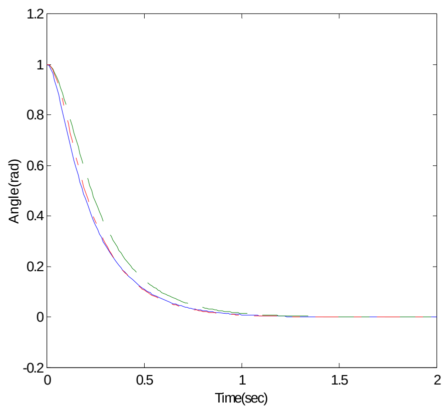

Fig.2 Response of x_1

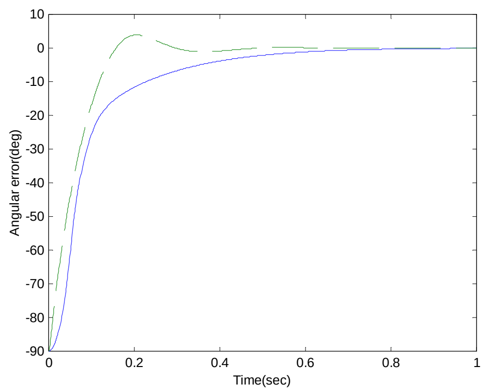

Fig.4 (b) position error of link 2

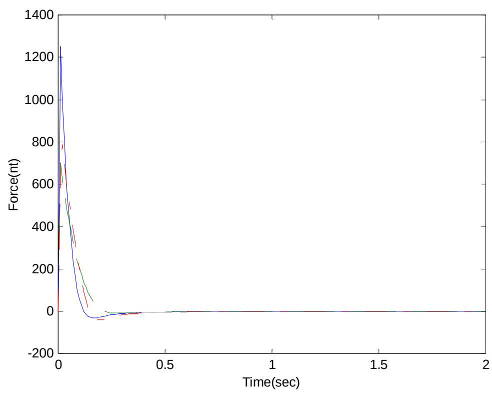

Fig.3 Response of $u(t)$

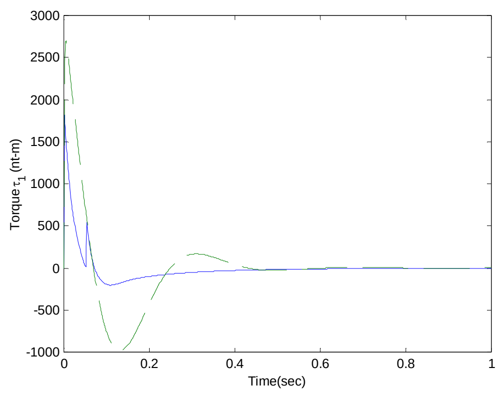

Fig.5 (a) Control torque of link 1

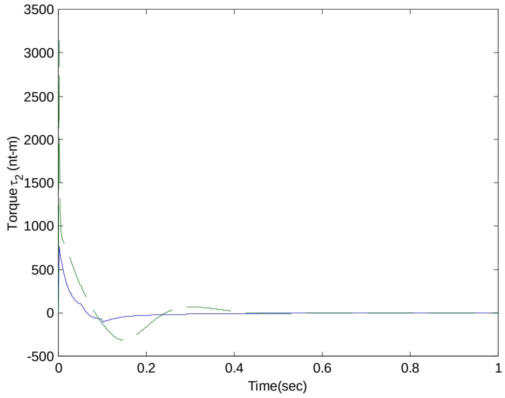

Fig.5 (b) control torque of link 2.

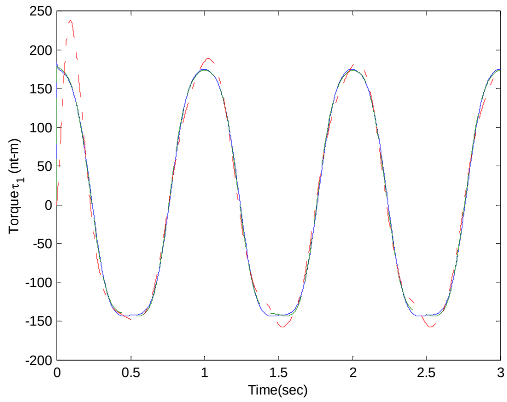

Fig.7 (a) Control torque of link 1.

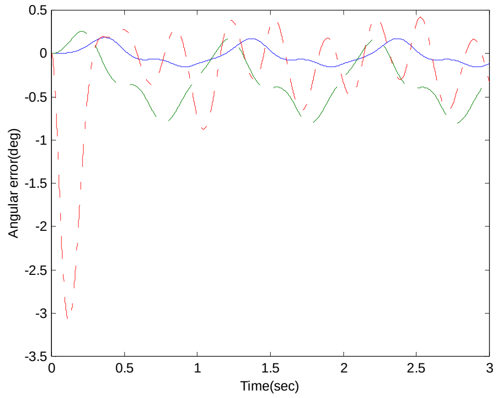

Fig.6 (a) Position error of link 1.

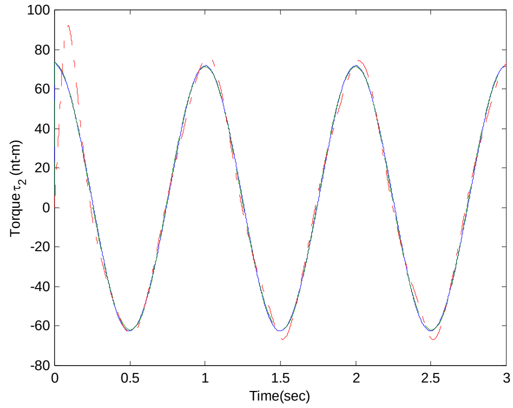

Fig.7 (b) control torque of link 2.

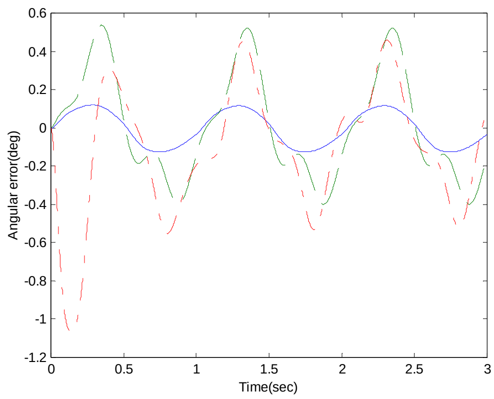

Fig.6(b) position error of link 2.

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多變數非線性系統應用 **Takagi-Sugeno** 模糊模式設計之適應控制方法

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摘　要

本文對於多變數非線性系統提出一應用 Takagi-Sugeno 模糊模式設計之適應控制方法。系統根據 Lyapunov 的線性化理論先求出動作範圍的區域線性數學模式，再以模糊推論的方法導出 T-S 的模糊模式，當此系統符合所設定的條件時，模糊模式即可解析成一個擾動的線性系統，而所推導的適應滑動模式控制法則可以確保全域漸進穩定的控制成效。相較於一般的 PDC 控制結構，本文所提出的方法較為簡單且能應用於實體控制。經由電腦模擬的結果，顯示此理論之正確性與實用性。

關鍵字：Takagi-Sugeno 模糊模式、適應滑動模式控制法。

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